THE RISK OF STRATEGIC INVESTMENT IN THE PSSR AUCTION

A report prepared for Three regarding the upcoming auction for 2.3GHz and 3.4GHz spectrum

NON CONFIDENTIAL

January 2017

CONTENTS

Executive Summary 4
Context 4
Intrinsic values 4
Strategic investment 4
Our approach 6
Results 6

1 Introduction 7
1.1 Objectives of this report 7
1.2 Strategic investment 7
1.3 [✂-Why the auction is important for the target(s).] 10
1.4 Structure of this report 10

2 Approach to assessing intrinsic and strategic value 11
2.1 Overview 11
2.2 [✂] 11
2.3 Assumptions and approach to estimating intrinsic and strategic value 12
2.4 Summary 12

3 Key results 13
3.1 Results under our central assumptions 13
3.2 Results with sensitivities 13
3.3 Conclusion 13

Annexe - Input data used 14

Annexe – Differentiated Bertrand modelling 15
Overview of modelling 15
Deriving the demand parameters 16
Estimating the updated prices and associated volumes 17

EXECUTIVE SUMMARY

Context

In November 2016, Ofcom published a consultation[1] on competition measures and specific aspects of auction design for the upcoming public sector spectrum release (PSSR) auction of 2.3GHz and 3.4GHz spectrum. Both 2.3GHz and 3.4GHz[2] can be used for 4G services. In addition, 3.4GHz is, in Ofcom's view likely to be one of the main bands that will be used for 5G services.

Ofcom is proposing to impose an overall spectrum cap of 40% on immediately usable spectrum. This means that BT/EE cannot bid for any of the 2.3GHz spectrum. However, as Ofcom considers that 3.4GHz spectrum is not immediately usable, BT/EE is free to acquire part or all of the 3.4GHz, as Ofcom's spectrum cap does not apply to this band.

In this context, Three has commissioned Frontier Economics to [✂-consider the size of incentives to bid strategically and the risk that the incentives are large enough to affect the outcome of the auction for 2.3 and 3.4GHz spectrum.][3]

Intrinsic values

Market mechanisms, such as auctions, are used by authorities to allocate spectrum between potential users on the assumption that this will result in an efficient outcome. In particular, well designed auctions can ensure that spectrum is allocated to potential users who will use the spectrum to lower the costs of delivering services to end users (productive efficiency) and to innovate in the delivery of services to end users (dynamic efficiency).

Such an outcome can occur where bidders bid based on the 'intrinsic' value of spectrum. Ofcom has defined the intrinsic value as:

"The present value of additional profits a bidder expects to earn when holding the spectrum compared to not holding it - in the absence of any strategic considerations to obtain spectrum that reduces competition in mobile services from the existing level."[4]

Strategic investment

As spectrum is a scarce resource, investment in spectrum will reduce the amount available to other users, which in turn could affect their ability to compete. Ofcom has defined the strategic investment value as:

---

1 Ofcom (November 2016) - Public Sector Spectrum Release (PSSR) - Award of the 2.3 GHz and 3.4 GHz bands

2 3.4GHz could also be used for backhaul.

3 [✂]

4 Ofcom (November 2016) - Public Sector Spectrum Release (PSSR) - Award of the 2.3 GHz and 3.4 GHz bands - paragraph 4.162.

"The present value of additional expected profits earned from bids that affect the future structure of competition in mobile services by depriving one or more competitors of spectrum."[5]

Strategic investment value may be generated in two ways:

- By restricting the ability of the target of strategic bidding to serve customers, leading them to reduce demands on their network by charging higher prices than it otherwise would, leading to increased churn and reduced acquisition; and
- By restricting the target's ability to offer innovative services, leading to increased churn or reduced ability to compete in a segment of the market.

Strategic investment value will therefore be associated with a risk of worse outcomes for consumers e.g. increased prices and/or reduced innovation.

We have been asked to consider the likelihood of an outcome where strategic investment could prevent an efficient allocation of spectrum for 2.3GHz and/or 3.4GHz. Therefore, we have considered a case where (i) absent any strategic investment considerations, the efficient outcome would be for the target of strategic investment to obtain the spectrum[6], but (ii) the existence and significance of strategic value means that the target is denied spectrum that it values (intrinsically) more highly than the strategic investor.

This is more likely where the benefits for a strategic bidder from denying spectrum to the target are relatively high, for example when the strategic investor has a high market share over which it will benefit from any increase in prices across the market (or a particular segment of the market) and it will gain a high proportion of customers leaving the target network. [✂][7].

Given the focus of our analysis, we assume that absent strategic bidding, [✂-the target(s)]has a higher intrinsic value for at least some of the spectrum than the other operators. Therefore, in our analysis, absent strategic bidding, [✂-the target(s)]would acquire some of the spectrum on offer and this would likely be the efficient outcome.

In total, there is 40MHz of 2.3GHz and 150MHz of 3.4GHz available in the PSSR auction and four existing mobile network operators. Strategic investment by [✂-the perpetrator(s)]would not require [✂-the perpetrator(s)] to acquire all blocks available, but to bid in a way that would deprive [✂-the target(s)]of at least an increment of spectrum which it would otherwise acquire and which would diminish [✂-the target(s)]'s ability to compete.

Ofcom has itself acknowledge that there is a material risk of strategic bidding, although it considers that this risk mainly relates to 2.3GHz:

"The MNOs with smaller shares of spectrum have an opportunity to bid for the 2.3 GHz spectrum themselves. However, operators with large spectrum shares may have a higher valuation for the spectrum not because they would use it more

[5] Ibid.

[6] e.g., Where the intrinsic value of spectrum for the 'target' of strategic bidding is higher than the corresponding value for the strategic investor, assuming the valuation of other parties would be lower.

[7] [✂]

effectively, but because competition in the mobile market would be weaker if they acquired it. The fact that there is only a relatively small amount of spectrum in the 2.3 GHz band may make this kind of strategic investment more likely."[8]

Our approach

To assess the impact of [✂-the perpetrator(s)] bidding strategically to deprive [✂-the target(s)]of spectrum, we have used a model of competition similar to that used by the European Commission to assess the possible price increases following mergers between mobile operators (a differentiated Bertrand model). We have evaluated two effects resulting from [✂-the target(s)]failing to acquire sufficient spectrum (where this could mean [✂-the target(s)]fails to acquire any spectrum or acquires insufficient spectrum to avoid adverse consequences on its cost or market position):

- [✂-The adverse consequences anticipated.]

[✂-More detail on the two adverse consequences considered]

[✂][9][✂][10][✂][11]

[✂][12][✂][13]

[✂]

Our modelling

[✂-Details of the modelling used.]

Results

[✂-The results of the modelling.][14] [✂]

[8] Ofcom (November 2016) - Public Sector Spectrum Release (PSSR) - Award of the 2.3 GHz and 3.4 GHz bands paragraph 1.24.

[9] [✂]

[10] [✂]

[11] [✂]

[12] [✂]

[13] [✂]

[14] [✂]

1 INTRODUCTION

This section is structured as follows:

- In section 1.1, we set out the objectives of this report;
- In section 1.2, we explain the conditions under which strategic bidding may occur;
- In section 1.3, we describe the importance of the spectrum auction to [✂-the target(s)]; and
- In section 1.4, we set out the structure for the rest of the report.

1.1 Objectives of this report

[✂][15] [✂][16].

[✂]

Furthermore, the focus of our report is to consider the likelihood of an outcome where strategic investment could distort an otherwise efficient spectrum allocation result. This effectively reflects the risk of strategic investment identified by Ofcom that:

"[e]ven if an operator has a higher intrinsic value for some spectrum than other bidder(s), it may fail to acquire the spectrum in the auction if it is the victim of strategic investment by another operator(s). In this situation, we would expect consumers to be made worse off by the spectrum going to the highest bidder in the auction, because competition would be weaker"[17]

In other words, we are considering the likelihood of a scenario where, absent any strategic investment considerations, the efficient outcome would be for the target of the strategic investment to obtain sufficient spectrum. The existence and significance of strategic value, however, means that the target is denied spectrum which it values (intrinsically) more highly than the strategic investor. Specifically, in this report we consider the likelihood of strategic bidding for spectrum that would have a material impact on [✂-the target(s)]'s costs and ability to compete in the market.

1.2 Strategic investment

Market mechanisms, such as auctions, are used by authorities to allocate spectrum between potential users on the understanding that this will result in an efficient outcome. In particular, well designed auctions can ensure that spectrum is allocated to potential users who will use the spectrum to lower the costs of delivering services to end users (productive efficiency) and to innovate in the

15 [✂]

16 [✂]

17 Ofcom (November 2014) - Public Sector Spectrum Release (PSSR) - Award of the 2.3 GHz and 3.4 GHz bands - paragraph 7.100

delivery of services to end users (dynamic efficiency). Such an outcome can occur where bidders bid based on the 'intrinsic' value of spectrum.

In this report, we define capacity as meaning both i) the total amount of data that can be transferred over a given period ("bandwidth") in a given cell and ii) the average user speeds during the busy hour as the available bandwidth is shared across the active users in the cell.

The maximum amount that an operator should be willing to pay for capacity spectrum should be equal to the increase in its net present value from having the spectrum for providing capacity relative to not having the spectrum. The value of spectrum can be split up into different components:

- **Intrinsic value**. Ofcom has defined the intrinsic value as:

  "The present value of additional profits a bidder expects to earn when holding the spectrum compared to not holding it - in the absence of any strategic considerations to obtain spectrum that reduces competition in mobile services from the existing level."[18]

  The intrinsic value of spectrum can be estimated as the sum of the net present value of the extra profits that an operator can make due to a reduction in costs and/or opportunities to increase revenues by offering differentiated or more advanced services as a result of acquiring that spectrum – without any change in the competitive conditions of the retail market.

  In the absence of acquiring more spectrum, an operator will make a profit-maximising decision in two dimensions:

  a. the extent to which it will try to match the quality of service of other operators by investing in additional network capacity through additional equipment (and incurring higher costs – the avoidance of which is technical value); and

  b. the extent to which it will accept a quality of service reduction or target a lower volume of customers in order to maintain performance for remaining customers (the avoidance of which can be considered commercial value).

  The combination of these two values is the relevant value for consideration within our analysis.

- **Strategic investment value**. Ofcom has defined strategic investment value as:

  "The present value of additional expected profits earned from bids that affect the future structure of competition in mobile services by depriving one or more competitors of spectrum."[19]

  There are different ways in which an operator may benefit from depriving another operator of spectrum:

[18] Ofcom (November 2016) - Public Sector Spectrum Release (PSSR) - Award of the 2.3 GHz and 3.4 GHz bands - paragraph 4.162.

[19] Ibid.

- By restricting the ability of the target of strategic bidding to serve customers, leading them to reduce demands on their network by charging higher prices than they otherwise would, leading to increased churn and reduced acquisition; and
- By restricting the target's ability to offer innovative and/or high quality services (e.g. high speed LTE services or 5G services) leading to increased churn or a reduced ability to compete in a segment of the market.

Strategic investment value will therefore be associated with a risk of worse outcomes for consumers e.g. increased prices and/or reduced innovation.

1.2.1 Conditions for strategic bidding to be problematic

We have been asked to consider the likelihood of an outcome where strategic investment could prevent an efficient allocation of spectrum for 2.3GHz and/or 3.4GHz. Therefore, the conditions for this to occur imply that:

(i) absent any strategic investment considerations, the efficient outcome would be for the target of strategic investment to obtain the spectrum[20], but

(ii) the existence and significance of strategic value means that the target is denied spectrum that it values (intrinsically) more highly than the strategic investor.

This is more likely where the benefits for a strategic bidder from denying spectrum to the target are relatively high. This is likely to occur when the strategic investor has a high market share over which it will benefit from any increase in prices across the market (or a particular segment of the market) and it will gain a high proportion of customers leaving the target network. [✂][21].

Given the focus of our analysis, we assume that absent strategic bidding, [✂-the target(s)]has a higher intrinsic value for at least some of the spectrum than the other operators[22]. Therefore, in our analysis, absent strategic bidding, [✂-the target(s)]would acquire some of the spectrum on offer and this would likely be the efficient outcome.

For successful strategic investment to occur, it is not necessary for the strategic value component to be large, although this may increase the overall likelihood of strategic investment. Instead it is sufficient that the strategic value is large enough to mean that [✂-the perpetrator(s)]'s total value is pushed over the amount of [✂-the target(s)]'s intrinsic value.

---

20 I.e. The intrinsic value of spectrum for the 'target' of strategic bidding is higher than the corresponding value for the strategic investor, assuming the valuation of other parties would be lower..

21 [✂]

22 We are aware of a related argument, that was an issue in the design of the 800MHz/2.6GHz auction, that absent a minimum amount of spectrum, Three/a smaller mobile operator may not be a viable competitor. In such a context, a socially optimal outcome may not be achieved even if operators bid based on intrinsic values, as smaller operators may have a lower commercial value of spectrum. This could also justify intervention to ensure that such spectrum is not denied to Three/a smaller operator. Whilst this may also be relevant in the forthcoming PSSR auction, it is not an issue we have considered in this report.

In total, there is 40MHz of 2.3GHz and 150MHz of 3.4GHz available in the PSSR auction and four existing mobile network operators. Strategic investment by [✂-the perpetrator(s)]would not require [✂-the perpetrator(s)]to acquire all blocks available, but to bid in a way that would deprive [✂-the target(s)]of at least an increment of spectrum which it would otherwise acquire and which would diminish [✂-the target(s)]'s ability to compete.

Ofcom has itself acknowledge that there is a material risk of strategic risk, although it considers that this risk mainly relates to 2.3GHz:

"The MNOs with smaller shares of spectrum have an opportunity to bid for the 2.3 GHz spectrum themselves. However, operators with large spectrum shares may have a higher valuation for the spectrum not because they would use it more effectively, but because competition in the mobile market would be weaker if they acquired it. The fact that there is only a relatively small amount of spectrum in the 2.3 GHz band may make this kind of strategic investment more likely."[23]

1.3 [✂-Why the auction is important for the target(s).]

[✂][24]

[✂[25]

- [✂]

Figure 1. [✂]

[✂]

[✂]

Figure 2. [✂]

[✂]

1.4 Structure of this report

The rest of this report is structured as follows:

- Section 2 explains our approach; and
- Section 3 presents our key results.

23 Ofcom (November 2016) - Public Sector Spectrum Release (PSSR) - Award of the 2.3 GHz and 3.4 GHz bands paragraph 1.24.

24 [✂]

25 [✂]

2 APPROACH TO ASSESSING INTRINSIC AND STRATEGIC VALUE

2.1 Overview

- [✂][26]
- [✂]

2.2 [✂]

[✂][✂][27]

[✂][28] [✂][29] [✂][30].

[✂]

[31][✂]

Figure 3. [✂]

[✂]

[✂][32]

[✂][33]

[✂][34]

- [✂][35]
- [✂][36]

[✂][37]

26 [✂]

27 [✂]

28 [✂]

29 [✂]

30 [✂]

31 [✂]

32 [✂]

33 [✂]

34 [✂]

35 [✂]

36 [✂]

37 [✂]

[✂][38]

[✂][39] [✂][40] [✂][41] [✂][42]

2.3 Assumptions and approach to estimating intrinsic and strategic value

[✂][✂][43]

[✂][44]

[✂]

2.4 Summary

[✂]

Figure 4. [✂]

[✂]

[38] [✂]

[39] [✂]

[40] [✂]

[41] [✂]

[42] [✂]

[43] [✂]

[44] [✂]

3 KEY RESULTS

3.1 [✂]Results under our central assumptions

[✂]

Figure 5. [✂]
[✂]

[✂][45] [✂][46].

[✂][47].

3.2 Results with sensitivities

[✂]

Figure 6. [✂]
[✂]

[✂]

Figure 7. [✂]

3.3 Conclusion

[✂][48][✂]

45 [✂]
46 [✂]
47 [✂]
48 [✂]

ANNEX A- INPUT DATA USED

The following table summarises the sources and assumptions that we have used for the different input data.

Figure 8. Input data used

Variable	Data source / assumptions used	Values
Number of post-paid subscribers	Telegeography	Three – 6.2m BT/EE – 18.0m Vodafone – 13.6m O2 – 13.3m Virgin – 2.3m Tesco – 0.9m
Monthly post-paid ARPUs (excluding Handset revenues)	Analysis Mason	Three – £19.74 BT/EE – £27.53 Vodafone – £25.72 O2 – £27.77 Virgin – assumed to be equal to Three - £19.74 Tesco – assumed to be equal to Three - £19.74
Incremental margins	[✂]	[✂]
Diversion ratios	Calculated based on switching data from Kantar.	[✂]
[✂]	[✂]	[✂]
[✂]	[✂]	[✂]

Source: Frontier

ANNEX B– DIFFERENTIATED BERTRAND MODELLING

Overview of modelling

Differentiated Bertrand models rely on the assumption that firms profit maximise by determining prices, with the volume of product sold depending on the price set. For each product, it is possible to derive a demand curve, which shows how the demand for a given product depends on the price of both that particular product (own-price effects) as well as the prices of other substitutable products (cross-price effects). By assuming that the demand curves are linear, it is possible to estimate how the demand for a product will change in response to a price change for that product, based on the following parameters:

- Retail prices;
- Marginal costs;
- Volumes; and
- The ownership of other products.

The own price effect (how the demand for a firm's product changes when it varies its prices) will be lower the further prices are above marginal costs, as the price-cost margin is a sign of market power[49]. The cross-price effects can be estimated based on:

- The own price effects; and
- Diversion ratios.

Diversion ratios reflect consumers' second choice of product if they were to stop using their current product due to a price increase. This is generally estimated by measuring which products consumers move to when they stop consuming a particular product[50]. The cross-price effect will be higher for those products that have high diversion ratios (i.e. where a large number of customers would shift between these products if prices for one product changes).

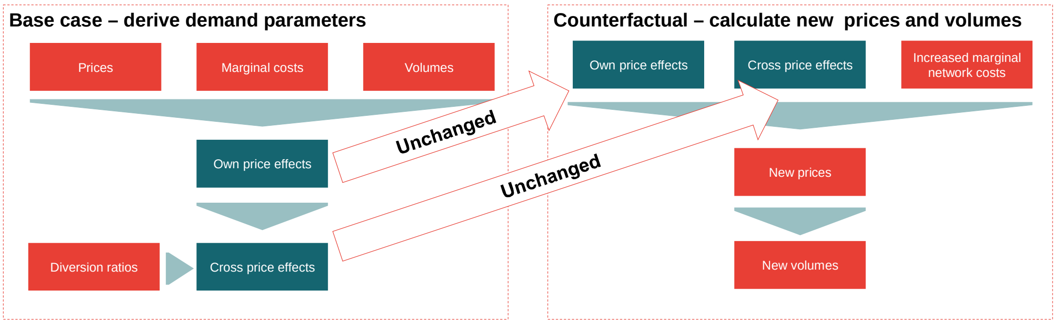
Once the own-price and cross-price effects have been estimated based on the above parameters (the "factual"), it is possible to assess the impact of increasing [✂] costs. A new equilibrium for the retail prices and volumes for all market players can be found [✂]. The following figure summarises how we have used a differentiated Bertrand model to estimate the impact of [✂] costs increasing.

---

49 This is essentially just the "Lerner" condition where the price mark-up is equal to the inverse of the own price elasticity

50 Equivalently, diversion ratios can also be viewed as a measure of where consumers come from when they start using a new product.

Figure 9. Price setting in differentiated markets



Source: *Frontier*

Deriving the demand parameters

- We are looking to find prices that maximise profits, as a function of quantities, ownership, costs, and demand curves.
- Profits of a firm are (π is profit, $\theta_{i,j}$ is ownership share of firm i in firm j, p_i and q_i are price and quantity for each firm where "W" stands for "wholesale"):

$$\pi_1 = \theta_{1,1}\pi_1 + \theta_{1,2}\pi_2 + \theta_{1,W}\pi_W \ldots + \theta_{1,n}\pi_n$$
$$= \theta_{1,1}(p_1 q_1 - c_1 q_1) + \theta_{1,2}(p_2 q_2 - c_2 q_2) + \theta_{1,W}(p_W q_W - c_W q_W) + \cdots + \theta_{1,n}(p_n q_n - c_n q_n)$$

- Take FOC w.r.t p_1 to maximise π_1 (assuming SOC holds):

$$\theta_{1,1}\left(q_1 + (p_1 - c_1)\frac{\partial q_1}{\partial p_1}\right) + \theta_{1,2}\left((p_2 - c_2)\frac{\partial q_2}{\partial p_1}\right) + \theta_{1,W}\left((p_W - c_W)\frac{\partial q_W}{\partial p_1}\right) + \cdots + \theta_{1,n}\left((p_n - c_n)\frac{\partial q_n}{\partial p_1}\right) = 0$$

- Similarly for p_2:

$$\theta_{2,1}\left((p_1 - c_1)\frac{\partial q_1}{\partial p_2}\right) + \theta_{2,2}\left(q_2 + (p_2 - c_2)\frac{\partial q_2}{\partial p_2}\right) + \cdots + \theta_{2,n}\left((p_n - c_n)\frac{\partial q_n}{\partial p_1}\right) = 0$$

- The model estimates the demand curve slope (elasticities) matrix $\beta \equiv \Delta\left(\underline{p}\right)$ and the demand curve intercept a:
  - Own-price elasticities: $\frac{\partial q_1}{\partial p_1} = -\frac{q_1}{(p_1 - c_1) - \Theta \cdot \mathrm{DR} \cdot (\underline{p} - \underline{c}) - \Theta_W \cdot \mathrm{DR} \cdot (\underline{p_W} - \underline{c_W})}$ (this comes from FOC from firm 1)
  - Cross-price elasticities: $\frac{\partial q_i}{\partial p_1} = \frac{\partial q_1}{\partial p_1}\frac{\partial q_i}{\partial q_1} = -\frac{\partial q_1}{\partial p_1} DR_{1,i}$
- Intercept: $\underline{\mathrm{a}} = \underline{\mathrm{q}} - \Delta\left(\underline{\mathrm{p}}\right) \cdot \underline{\mathrm{p}}$ (from linear demand)

Estimating the updated prices and associated volumes

- Rewriting all of the FOCs in matrix form:

$$\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{pmatrix} + \underbrace{\begin{pmatrix} \theta_{1,1} & \theta_{1,2} & \cdots & \theta_{1,n} \\ \theta_{2,1} & \theta_{2,2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{n,1} & \cdots & \cdots & \theta_{n,n} \end{pmatrix}}_{\equiv \Theta} \underbrace{\begin{pmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_1} & \cdots & \frac{\partial q_n}{\partial p_1} \\ \frac{\partial q_1}{\partial p_2} & \frac{\partial q_2}{\partial p_2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial q_1}{\partial p_n} & \cdots & \cdots & \frac{\partial q_n}{\partial p_n} \end{pmatrix}}_{\equiv \Delta(\underline{p})'} \begin{pmatrix} p_1 - c_1 \\ p_2 - c_2 \\ \vdots \\ p_n - c_n \end{pmatrix}$$

$$+ \underbrace{\begin{pmatrix} \theta_{1,W} & \theta_{1,W} & \cdots & \theta_{1,W} \\ \theta_{2,W} & \theta_{2,W} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \theta_{n,W} & \cdots & \cdots & \theta_{n,W} \end{pmatrix}}_{\equiv \Theta_W} \underbrace{\begin{pmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_1} & \cdots & \frac{\partial q_n}{\partial p_1} \\ \frac{\partial q_1}{\partial p_2} & \frac{\partial q_2}{\partial p_2} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial q_1}{\partial p_n} & \cdots & \cdots & \frac{\partial q_n}{\partial p_n} \end{pmatrix}}_{\equiv \Delta(\underline{p})'} \begin{pmatrix} p_W - c_W \\ p_W - c_W \\ \vdots \\ p_W - c_W \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

- Rewriting in vector notation:

$$\underline{q}\left(\underline{p}\right) + \Theta \cdot \Delta\left(\underline{p}\right)' \cdot \left(\underline{p} - \underline{c}\right) + \Theta_W \cdot \Delta\left(\underline{p}\right)' \cdot \left(\underline{p}_W - \underline{c}_W\right) = \underline{0}$$

- Solving for $\underline{p}$:

$$\Theta \cdot \Delta\left(\underline{p}\right)' \cdot \left(\underline{p} - \underline{c}\right) = -\underline{q}\left(\underline{p}\right) - \Theta_W \cdot \Delta\left(\underline{p}\right)' \cdot \left(\underline{p}_W - \underline{c}_W\right)$$

$$\Theta \cdot \Delta\left(\underline{p}\right)' \cdot \underline{p} = \Theta \cdot \Delta\left(\underline{p}\right)' \cdot \underline{c} - \underline{q}\left(\underline{p}\right) - \Theta_W \cdot \Delta\left(\underline{p}\right)' \cdot \left(\underline{p}_W - \underline{c}_W\right)$$

$$\underline{p} = \underline{c} - \left[\Theta \cdot \Delta\left(\underline{p}\right)'\right]^{-1} \cdot \underline{q}\left(\underline{p}\right) - \left[\Theta \cdot \Delta\left(\underline{p}\right)'\right]^{-1} \cdot \left[\Theta_W \cdot \Delta\left(\underline{p}\right)' \cdot (\underline{p}_W - \underline{c}_W)\right]$$

- But $\underline{q}\left(\underline{p}\right)$ on the RHS is still a function of $\underline{p}$! Use linear demand:

$$q_1 = a_1 + \frac{\partial q_1}{\partial p_1} p_1 + \frac{\partial q_1}{\partial p_2} p_2 + \cdots + \frac{\partial q_n}{\partial p_1} p_n$$

$$\underline{q}(\underline{p}) = \underline{a} + \Delta\left(\underline{p}\right) \cdot \underline{p}$$

- Substitute this expression for $\underline{q}\left(\underline{p}\right)$ in (1) to get

$$\underline{p} = \underline{c} - \left[\Theta \cdot \Delta\left(\underline{p}\right)'\right]^{-1} \cdot \left(\underline{a} + \Delta\left(\underline{p}\right) \cdot \underline{p}\right) - \left[\Theta \cdot \Delta\left(\underline{p}\right)'\right]^{-1} \cdot \left[\Theta_w \cdot \Delta\left(\underline{p}\right)' \cdot (p_w - c_w)\right]$$

- Multiply through (from the left) by $\Theta \cdot \Delta\left(\underline{p}\right)'$, and then solve for $\underline{p}$:

$$\left(\Theta \cdot \Delta\left(\underline{p}\right)'\right) \cdot \underline{p} = \left(\Theta \cdot \Delta\left(\underline{p}\right)'\right) \cdot \underline{c} - \left(\underline{a} + \Delta\left(\underline{p}\right) \cdot \underline{p}\right) - \left[\Theta_w \cdot \Delta\left(\underline{p}\right)' \cdot (p_w - c_w)\right]$$

$$\left[\left(\Theta \cdot \Delta\left(\underline{p}\right)'\right) + \Delta\left(\underline{p}\right)\right] \cdot \underline{p} = \left(\Theta \cdot \Delta\left(\underline{p}\right)'\right) \cdot \underline{c} - \underline{a} - \left[\Theta_w \cdot \Delta\left(\underline{p}\right)' \cdot (p_w - c_w)\right]$$

- Using the derived demand parameters from the previous section it's possible to estimate the update prices using the following equation by plugging in the updated marginal costs.

$$\underline{p} = \left[\left(\Theta \cdot \Delta\left(\underline{p}\right)'\right) + \Delta\left(\underline{p}\right)\right]^{-1} \left[\left(\Theta \cdot \Delta\left(\underline{p}\right)'\right) \cdot \underline{c} - \underline{a} - \left[\Theta_w \cdot \Delta\left(\underline{p}\right)' \cdot (p_w - c_w)\right]\right]$$

www.frontier-economics.com