Comments on Ofcom's Proposed Spectrum Auction: 2.6 GHz, 2010 MHz and 2290 MHz

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1. Introduction

Ofcom is proposing to award licenses for three spectrum bands: the 2.6 GHz band (divided into 38 blocks of 5 MHz), the 2010 MHz band (1 block of 15 MHz), and the 2290 MHz band (1 block of 10 MHz).[1] The three bands are either unused at present or expected to become available for new applications in 2007. Ofcom has identified four main categories of services for which prospective operators have indicated an interest in using these bands: (a) advanced mobile telephony services using 3G technologies and their evolutions; (b) broadband wireless services using WiMAX standards or a variant of the 3G family (UMTS TDD); (c) mobile multimedia services that could complement cellular or broadband wireless services or be stand-alone services; and (d) programme making and special events (PMSE) services, primarily for digital video applications.

The licenses awarded will be *tradable.* The proposed auction is intended to ensure "an efficient primary assignment", and in Ofcom's view subsequent spectrum trading will provide "additional scope for efficiency, through the opportunity for licensees to respond to changes in conditions over time" (Ofcom, 6.105).

The 38 blocks of the 2.6 GHz band can be aggregated into packages of ***paired lots*** consisting of two 5 MHz blocks each, and ***unpaired lots*** consisting of one 5 MHz block each. Because of technological constraints, there can be at most 14 paired lots, and between 9 and 38 unpaired lots. The single 15 MHz block of the 2010 MHz band will be treated (roughly) as two blocks of the 2.6 GHz band. The allocation of the blocks, and their division into paired and unpaired lots, will be determined by an auction process. The 2290 MHz band will be treated separately and allocated in a different auction.

Ofcom (see 6.155) is also considering setting a 'safeguard cap' at 90 MHz which would allow any one bidder to acquire at most 2 x 45 MHz of paired spectrum, 90 MHz of unpaired spectrum, or an equivalent combination. The precise way that a cap of this sort would apply across the different bands will depend upon the way the bands are grouped for auction.

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1 Ofcom, "Award of available spectrum: 2500-2690 MHz, 2010-2025 MHz and 2290-2300 MHz," 11 December 2006.

In the remainder of this note we first describe Ofcom's proposed auction rules as we understand them, and then consider some potential problems with the auction design which, in our view, suggest the need for a reconsideration of certain issues.

In summary, Ofcom's auction design is a modified version of the 'clock-proxy' auction which has been proposed in the recent literature. To our knowledge, clock-proxy auctions have never been used in practice, and neither have they been subject to comprehensive laboratory testing. Other types of multi-round combinatorial auction have been tested in laboratory settings, and two-stage Anglo-Dutch auctions are currently being used to sell virtual power plant capacity in Denmark. Ofcom's consultation document makes little or no reference to such alternative auction designs, however.

Given the importance of the auction for the 2.6 GHz and the 2010 MHz bands, we suggest that a number of alternative auction designs be given further consideration, and that they be subject to laboratory testing. Variants of Ofcom's proposed auction design should also be tested, since current theory alone is not sufficient to determine the merits and demerits of the detailed auction rules in combinatorial settings. Given that Ofcom is proposing to use a very similar auction design for the 1452-1492 spectrum band, the argument for laboratory testing is even stronger.

2. Proposed Auction Rules

Ofcom's proposal is to hold a separate auction for the 2290 MHz band, and a combined auction for the 2.6 GHz band and the 2010 MHz band. The proposed auction for the 2290 MHz band is a single-unit, sealed-bid, second-price auction to be run before the auction for the other two bands. The 2290 MHz band is not considered to be a substitute or a complement for the other two bands. We have not devoted any attention to this auction, so it is not discussed further here.

The second auction, for the 2.6 GHz and 2010 MHz Bands, is a multi-round combinatorial auction which will potentially consist of three stages:

1. An ascending, multi-round **Clock Stage**;
2. A (possible) **Best and Final Offers (BAFO) Stage**;
3. A final **Assignment Stage**.

We describe each in turn.

2.2 First Clock Stage - 2.6 GHz and 2010 MHz Bands

This is a multi-round ascending clock auction for three different ***lot types***: (i) paired lots of the 2.6 GHz band; (ii) unpaired lots of the 2.6 GHz band; and (iii) one lot of the 2010 MHz band.

In each round, the current ***clock price*** for each type of lot will be posted, and bidders will express their demands by specifying the number of each lot type desired at the current prices (subject to their eligibility determined by the auction activity rules). Each bid is considered to be for a 'package' of lots, so bidders cannot be awarded only a subset of the package on which they have bid in any

round. Each round's bids will remain binding on the bidders for the entire auction, but bids will be treated as mutually exclusive (so a single bidder cannot have more than one winning bid). After each round, the prices of lot types for which there is excess demand will be increased by a specified increment, while the prices of the lot types for which there is no excess demand will not change.

A clock auction differs from a standard simultaneous, multi-round ascending auction (SMRA) in that the auctioneer directly controls the pace of the auction and the movement of prices. That is, bidders do not choose the prices to bid for particular lots or packages; rather they simply express their demand for a single package for each set of posted prices as the auction proceeds.

Prices in the Clock Stage will start at a ***reserve price*** of £100,000 for a paired lot of the 2.6 GHz band and for the single lot of the 2010 MHz band, and £50,000 for an unpaired lot of the 2.6 GHz band. In any round, the price of a paired lot of the 2.6 GHz band will be set at twice the price of an unpaired lot of the 2.6 GHz band, so long as there is excess demand on both types of lot. But if total demand for unpaired lots is less than or equal to 9 (which is the minimum possible number of unpaired lots), while there is still excess demand for the paired lots, the price of unpaired lots will remain unchanged while the price of paired lots will increase. The price of the single lot of the 2010 MHz band is not linked to the prices of the other lot types.

Allowable bids in every round will be restricted by an ***activity rule***. In the first round, the number of lots on which a bidder can bid will depend upon its deposit. In any subsequent round, a bidder's eligibility will be determined by the bids made in the previous rounds. Specifically, *a bidder cannot increase the number of paired lots or unpaired lots on which it bids during the auction.* In this context, the single lot in the 2010 MHz band counts as equivalent to 2 unpaired lots of the 2.6 GHz band.

Consequently, in any round in which a bidder reduces its demand for any lot type, it reduces its eligibility to bid for that lot type in all subsequent rounds. In any round in which a bidder reduces its eligibility in this way, the bidder is permitted to make a ***best and final offer*** for all packages of lots for which it will not be allowed to bid thereafter. These bids cannot exceed the clock prices of the rounds in which they are made (***Capped BAFO bids***),[2] and are only taken into account in the BAFO Stage, if it takes place.

The Clock Stage ends when there is no excess demand for any type of lot. That is, when: (i) the total number of blocks of the 2.6 GHz band demanded by bidders is less than or equal to 37 or 38; (ii) the number of paired lots demanded is less than

[2] For example, suppose in Round 3 of the auction a bidder demands 3 paired and 5 unpaired lots of the 2.6 GHz band, and that after the prices of both types of lot increase in Round 4, the bidder reduces its demand to 1 paired and 3 unpaired lots. Then the bidder can submit best and final offers for all combinations containing 2 or 3 paired lots and 4 or 5 unpaired lots, so long as the prices specified by the bidder for these combinations do not exceed what they would cost at the current (Round 4) clock prices.

or equal to 14; and (iii) at most one bidder demands the single block of the 2010 MHz band.[3]

If the BAFO stage will not take place (see immediately below), then each bidder remaining in the final round of the Clock Stage will be allocated the package of lots on which it is bidding at the final clock prices. That is, if a bidder is demanding 3 paired and 5 unpaired lots of the 2.6 GHz band in the final round of the Clock Stage, then that bidder will be awarded 3 paired and 5 unpaired lots of the 2.6 GHz band at a price per lot determined by the final clock prices. The auction then proceeds directly to the Assignment Stage.[4]

2.3 Second Best and Final Offers (BAFO) Stage - 2.6 GHz and 2010 MHz Bands

If at the end of the Clock Stage demand is exactly equal to supply for each type of lot, then the auction will proceed directly to the Assignment Stage, and the BAFO stage will not take place. By contrast, if there is excess supply for any type of lot at the end of the Clock Stage, the auction will proceed to the BAFO Stage. The BAFO Stage is a combinatorial, sealed-bid auction that gives bidders an opportunity to make best and final offers on all packages of lots for which they are still eligible to bid.

Bidders remaining at the end of Clock Stage (i.e. those bidders who are still actively bidding at the Clock Stage's final round prices), will be able to make best and final offers on those packages for which they are still eligible to bid, with no restrictions on the amounts of these bids. That is, if a bidder is demanding 3 paired and 5 unpaired lots of the 2.6 GHz band in the final round of the Clock Stage, that bidder may then place ***uncapped BAFO bids*** for any combination of lots with weakly fewer than 3 paired and 5 unpaired lots.

Bidders who have reduced their eligibility during the auction (including those who have stopped bidding entirely), will have placed best and final offers on the packages for which they were eligible at various points during the Clock Stage. Ofcom will then consider *all* of the bids made during the Clock Stage and in the BAFO Stage, and determine the "set of bids of greatest total value", subject to the requirements:

(i) at most one of all of the bids made by any bidder is accepted; and

(ii) each lot is awarded only once.

The first requirement eliminates the so-called "exposure problem" for bidders, since a bidder can only ever win one of the entire packages it has bid on.

In the BAFO Stage, the prices charged to winning bidders are determined by a ***second-price rule***. This rule is described in a number of different ways in Ofcom's document. In Section 8:

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[3] See Ofcom, A8.177 – A8.182 for details.

[4] Prices for packages of lots determined in the Clock or BAFO stages are called ***base prices*** by Ofcom. In the Assignment Stage the bidder may ultimately pay more for the lots it has won in the Clock Stage, but need not do so.

"8.71 The auctioneer would compute the price to be paid by each individual winning bidder according to a 'second price' rule. The characteristics of this second price rule are that:

- *the total amount of money to be paid to Ofcom is minimised; but at the same time*
- *no losing bidder or combination of bidders (including combinations of losing and winning bidders) would, on the basis of their bids, be willing to pay more.*

8.72 The main benefit of this pricing rule would be that the winning bidders collectively would only have to pay as much as is necessary to beat the losing bidders. In consequence the incentive to 'shade' bids (that is to say to bid less than the spectrum is actually worth to the bidder) should be significantly lower than in the case of a 'pay what you bid' pricing rule."

In Annex 8 in A8.75:

> *"In the best-and-final-offers stage, the price to be paid by each winner for the package of lots that they have won is determined by a* ***second price rule****. Each bidder pays the minimum amount necessary such that no other bidder or group of bidders would be prepared to make a counter offer for lots that would be preferred by the group and give at least as much revenue. This ensures that those bidders remaining at the end of open bidding phase have incentives to make bids fairly close to their true values rather than to shade their bids, as would happen if they simply paid what they bid."*

(emphasis added). And in Annex 8 in A8.170:

> *"A8.170 Winning prices for the first stage are determined using a second price rule. These are prices such that:*
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> - *there is no dissatisfied bidder or coalition of bidders able to suggest an alternative outcome (in terms of prices paid and lots received) preferred by all members of the coalition and leaving the seller no worse off;*
> - *these are the lowest such prices subject to revenue being at least as great as the outcome of the clock auction."*

(emphasis added). While not necessarily inconsistent, these descriptions leave us unclear as to what prices winning bidders will actually pay in the BAFO Stage, an issue discussed further in Section 3 below.

2.4 Third Assignment Stage - 2.6 GHz and 2010 MHz Bands

The first two stages of the auction determine: (i) the total number of paired and unpaired lots that are awarded; (ii) the winning bidders and the number of paired and unpaired lots that each winning bidder is awarded; and (iii) the base prices that winning bidders must pay for the packages of lots they have been allocated.

The Assignment Stage determines the ***specific frequencies*** within the 2.6 GHz band that each winning bidder will be assigned, and the final package prices. In

this stage, winning bidders are able to submit sealed ***top-up bids*** for the frequency ranges they prefer. These frequency ranges are restricted to ensure that (almost) all bidders will receive contiguous packages of spectrum.

The Assignment Stage proceeds with Ofcom creating a list, or menu, of all possible assignments of the winning bidders to particular frequencies, subject to the contiguity restriction. Bidders then place separate sealed top-up bids on each assignment in the menu (where placing a zero bid on one or all of the assignments is always possible). Ofcom will then determine the combination of top-up bids that maximises the total amount bid, subject to accepting at most one bid from each bidder, and awarding each frequency at most once. Winning bidders pay their top-up bids in addition to the prices specified at the end of the first two stages.

3. Issues with the Proposed Auction Design

There are a number of issues with the current specification of the auction which in our view deserve further consideration, and which we now discuss in turn.

3.1 Activity Rule in the Clock Stage

We assume that the purpose of the Clock Stage is the usual one of price discovery. The use of a multi-round, ascending auction should help to reduce "common-value uncertainty," enabling bidders to bid more aggressively with less fear of the "winner's curse." In addition, when interrelated products are being sold simultaneously, as is the case here, the information revealed in an ascending auction enables bidders to make better-informed decisions about the quantities of each lot type to demand, because it provides tentative price information on all lot types. This is especially important if the different lot types may be substitutes or complements for each other, as bidding in the absence of price information makes the problem of determining how much bidders are willing to bid on each possible package of lots much more difficult.[5]

However, with these considerations in mind, the proposed activity rule which prevents bidders from treating paired and unpaired lots as substitutes during the auction may be too restrictive. If a bidder's valuations for the different lot types are not independent – i.e. if different lot types are complements or substitutes – an overly restrictive activity rule has the potential to reduce the efficiency of the spectrum allocation.

Consider, for example, a bidder who is willing to substitute paired lots with unpaired lots if the price of paired lots increases relative to the price of unpaired lots. Under the proposed activity rule, the bidder is not permitted to increase its demand for unpaired lots when the price of paired lots increases, even if the bidder reduces its overall demand.

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[5] The case for dynamic auctions is further strengthened when it is recognized that it can be costly for bidders to determine their preferences in combinatorial settings. A dynamic auction, by providing tentative price information, helps to reduce these costs for bidders. See for instance, Ausubel and Cramton (2004) on this.

Ofcom appears to exclude the possibility that bidders may wish to switch between paired and unpaired lots during the auction (e.g. in 7.20 and A8.102), but it is not entirely clear on what basis it does so. For example, four of the current mobile operators obtained a single 5 MHz block of TDD spectrum in the 2000 auction, and this could potentially be 'paired' with a single block of unpaired TDD spectrum won in the 2.6 GHz auction. Moreover, Ofcom itself suggests that unpaired blocks of the 2.6 GHz band could be paired with spectrum of the 2010 MHz band (see 8.16). In these cases, bidders may indeed consider unpaired blocks in the 2.6 GHz band as substitutes for paired blocks in the 2.6 GHz band.

In addition, at least one mobile operator uses both FDD and TDD spectrum in different businesses,[6] so may wish to demand both types of spectrum in the auction. If bidders demand both types of lot, even if they are not substitutes in their preferences, budget constraints can make them so in practice. To see this, consider a bidder who wishes to buy one paired (P) and one unpaired (U) lot for different business purposes, and has the valuations:

$$V(P) = £100;\ V(U) = £40; \text{ and } V(P,U) = £140.$$

The bidder's overall budget constraint in the auction is £80. Suppose that at prices of £20 for unpaired lots and £40 for paired lots, demand for unpaired lots falls below 9, so the price of unpaired lots does not increase further. The bidder maintains demand for both types of lot until the price of the paired lots reaches £60, at which point its budget constraint binds and its profit-maximising bid is to bid for a single paired lot only. But if the price of a paired lot then reaches £80, the bidder would like to switch its demand to one unpaired lot, but cannot do so because of the activity rule. As a consequence the bidder drops out the auction altogether, even though there may be excess supply for unpaired lots, and it is willing to purchase one at the current price.

To overcome these potential problems, Ofcom might consider allowing bidders more flexibility in changing their bids between rounds. There seems to be little reason that bidders should not be allowed to switch demand between lot types subject to an appropriate eligibility rule, and possibly even to increase demand for one lot type more than they reduce their demand for a different lot type.

For example, Ofcom could require that bidders do not increase the total number of 5 MHz blocks on which they are bidding, as in the activity rule that has been suggested for the 1452-1492 MHz auction.[7] Alternatively, it could allow bidders even more flexibility subject to a "revealed-preference" activity rule.[8]

3.2 The BAFO Stage

The purpose of the BAFO Stage is less clear. It may be intended to generate additional price competition; or to encourage participation by weak bidders; or to give bidders a chance to fully express their willingness' to pay for packages of lots,

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6 http://news.zdnet.co.uk/communications/0,1000000085,39188192,00.htm

7 Ofcom, "Discussion document on the award of available spectrum 1452-1492 MHz: Auction design," 15 February 2007.

8 See Ausubel, Cramton and Milgrom (2005).

thus favouring a more efficient allocation. Numerous statements in Ofcom's document suggest the latter interpretation, which is also the justification typically given for the 'proxy' stage of a 'clock-proxy' auction.[9] However, it is unclear whether the particular design of the BAFO Stage manages to efficiently achieve any of these objectives.

When should the BAFO Stage take place?

In the first place, the BAFO Stage only takes place if there is excess supply for at least one lot type at the end of the Clock Stage. If the purpose of the BAFO Stage is to induce an *efficient allocation* of spectrum, by allowing bidders an opportunity to express their willingness to pay for packages of lots on which they did not have an opportunity to bid during the Clock Stage, then the BAFO stage should always take place. This suggests that the purpose of the BAFO stage may be primarily to eliminate excess supply. However, if this is the case then simpler methods for eliminating excess supply might be used, such as intra-round bidding, as proposed in Ausubel and Cramton (2004) and Ausubel, Cramton and Milgrom (2005).

Capped BAFO bids

Secondly, bidders should in principle be allowed to sincerely express their preferences on as many packages of lots as they are interested in buying, in order to fully express complementarities among, and substitution possibilities between, different lot types. Ofcom's proposed rules for the BAFO Stage bids may again be overly restrictive in this respect, and can actually prevent truthful bidding in both the Clock and BAFO stages.

For example, suppose a bidder's valuations for single units of two products, X and Y, are given by V(X) = £300 and V(X,Y) = £360, and prices in the clock auction increase in increments of £25. At prices for X and Y of (£50, £50), the bidder's profit-maximizing bid is for one unit of each product. When prices increase to (£75, £75) however, its profit-maximising bid is for one unit of X and zero of Y. At that point, if the bidder stops bidding for product Y, then under Ofcom's rules it can make a Capped BAFO bid for the combination (X,Y) of at most £150, although its actual valuation is £360. Therefore, by submitting profit-maximising bids, the bidder will never have a chance to express its full valuation for the two products. The bidder's best strategy may even be to keep bidding for (X,Y) as prices increase in the clock auction (i.e. by not submitting profit-maximising bids), in order to maintain bid flexibility in the BAFO stage. In any event, bids are likely to be distorted away from the bidder's true valuations, which has the potential to make the auction inefficient.

While Ofcom indicates that it does not necessarily believe that there will be significant demand for combinations of paired and unpaired lots (see 7.20 and A8.102), in the example presented in its slide presentation[10] at least one bidder is assumed to value such combinations, and the BAFO Stage is described as needed,

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[9] Ausubel and Milgrom (2002); also Ausubel, Cramton and Milgrom (2005).

[10] "Consultation proposals for the award of the spectrum bands 2500-2690 MHz, 2010-2025 MHz and 2290-2300 MHz", Riverside House, London 8 February 2007.

"to find the most efficient way to 'pack' the demands of different bidders, particularly since bidders may value combinations across different types of lot – this requires information about all of the combinations that they might be interested in buying, and not just those revealed through the clock auction."

We therefore suggest that consideration be given to modifying the rules on BAFO bids so that:

(i) Bidders make their Capped BAFO bids only after the Clock Stage is completed, when the price discovery process is complete. At that point, bidders are likely to have better information on which packages to bid on, and on what prices to offer for them.[11]

(ii) Capped BAFO bids should only be restricted so as to be consistent with bids made during the Clock Stage, i.e. by the 'revealed preferences' of the bidders as determined from their Clock Stage bids.[12]

Strategic bidding and the second-price rule

The third problem with the BAFO Stage auction is that the outcome of the proposed **second-price rule** is somewhat ambiguous, and evidently guarantees ***neither*** that all lots will be sold, nor that bidders will not wish to manipulate the outcome of the auction by bidding strategically. To see this, consider the following simple example. Suppose there are 7 blocks on sale and three bidders A, B, and C. Bidder C is only interested in winning 7 unpaired blocks, and has a valuation of £700 for this package, while bidders A and B are interested in winning 2 blocks each. Suppose that in the BAFO Stage the 'sincere' bids of the three bidders are the following:

1. Bidder A bids £500 for 2 unpaired lots;
2. Bidder B bids £500 for 1 paired lot;
3. Bidder C bids £700 for 7 unpaired lots.

These bids could result when, for example, bidder C drops out of the Clock Stage when prices reach £100 for unpaired lots, and the Clock Stage ends with excess supply. Bidders A and B then place their higher bids in the BAFO stage.

Given these bids, the proposed rule assigns 2 blocks each to bidders A and B, and requires them to pay a total price of £700, so that bidder C is not prepared to make

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[11] Ofcom have incorporated this rule in the proposed auction for the 1452-1492 spectrum band ("Discussion document on the award of available spectrum 1452-1492 MHz: Auction design," 15 February 2007).

[12] In the example specified above, this would imply that the BAFO bid on the combination (X,Y) is restricted by: $V(X) + £50 \leq V(X,Y) \leq V(X) + £75$, so a 'sincere' bid of $V(X,Y) = £360$ remains possible. A version of this rule has been proposed in Ausubel, Cramton and Milgrom (2005), Section 5.4.1.

a 'counter-offer' for 7 blocks.[13] However, the rule does not specify the exact price to be paid by each winning bidder. For example, a price of £500 for Bidder B and £200 for Bidder A appears to be consistent with the rule, as does a price of £350 for each bidder.

Although Ofcom does specify (in A8.172) that:

> *"there are many possible prices satisfying [the second-price rule]. Amongst all these possible prices, those closest to the clock auction outcome would be selected"*

this does not necessarily resolve the problems. First, the Clock Stage bids give prices of (just under) £100 per block, yielding a total price of (just under) £400, hence there is still some ambiguity concerning how the remaining £300 should be allocated.

Moreover, in examples of this type, there are incentives for bidders to strategically manipulate their bids, and the second-price rule does not eliminate these incentives. For example, if Bidder A submits a BAFO bid of £201 for 2 unpaired lots (instead of £500), then it can only be charged £201 for its 2 unpaired lots and all of the remaining £499 is paid by Bidder B.[14] Although this is an equilibrium, in the presence of 'strategic uncertainty' Bidder B may equally wish to manipulate its bid, by bidding £201 for a paired lot in the hope that Bidder A will bid truthfully (which is also an equilibrium). When both bidders A and B engage in this type of strategic manipulation, the auction can easily result in an inefficient allocation.[15]

Since truthful bidding is not an equilibrium in general, bidders will have incentives to bid strategically in order to manipulate prices, in both the Clock and BAFO stages of the auction.[16] Hence it is far from clear that an efficient, or even near-efficient, outcome can be expected.

Linkage between the Clock and BAFO stages

A further set of issues relates to the way in which the Clock Stage and the BAFO Stage are linked together by the auction rules. Ofcom's rule in A8.170 states that,

> "w*inning prices … are the lowest such prices subject to revenue being at least as great as the outcome of the clock auction."* [17]

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[13] The total price to be paid by A and B would presumably be less if reserve prices have been placed on the unsold blocks, i.e. the unsold blocks would then be treated as 'sold' to Ofcom for their reserve prices.

[14] Given Bidder A's BAFO bid of £201, note that Bidder B wants to bid £500 for 1 paired lot, so is still willing to bid truthfully.

[15] Typically, there are also mixed strategy equilibria in examples of this type which result in inefficient allocations with positive probability.

[16] If we alter the example above so that Bidder B values a single paired lot at £700, then Bidder A can force Bidder B to pay £699 for its lot, while it pays £1 for 2 unpaired lots, by dropping out of the Clock Stage at a price £1 per block say, and then submitting a Capped BAFO bid of £1 for 2 unpaired lots. Given this strategy of Bidder A, Bidder B can do no better than to continue to bid truthfully.

[17] Ofcom's rule in A8.170 does not fully specify how we should compute the revenue resulting from the Clock Stage. Is it the revenue derived from accepting the bids made in

This can come into conflict with the second-price rule, and create further incentives for strategic bidding. To see this suppose again that there are 7 blocks on sale but that the bidder's preferences are given by:

1. Bidder A wants 2 unpaired lots for a maximum price of £200;
2. Bidder B wants 1 paired lot for a maximum price of £200;
3. Bidder C wants 6 unpaired lots for a maximum price of £700.

In the absence of the rule in A8.170, with sincere bidding the Clock Stage ends when Bidders A and B drop out at a price per block of £100. The BAFO Stage then awards 6 unpaired lots to Bidder C for a price of £400. Ofcom's rule, however, implies that Bidder C must pay £600 for his lots, since this was the 'outcome' of the Clock Stage. Given this, Bidder C is better off reducing its demand and ending the Clock Stage when the price per block reaches say, £67, and then submitting a "Capped BAFO" bid of £401 for 6 unpaired lots. It thus obtains the 6 lots for £401 rather than £600.[18] Truthful bidding is not an equilibrium.[19]

In addition, even without the rule in A8.170, the rule which states that the BAFO Stage will only occur if the Clock Stage ends with excess supply can create similar incentives for manipulation. To see this, assume that prices in the BAFO Stage are not linked to Clock Stage prices, and consider the preferences:

1. Bidder A wants 2 unpaired lots for a maximum price of £200;
2. Bidder B wants 1 paired lot for a maximum price of £200;
3. Bidder C wants 7 unpaired lots for a maximum price of £900.

With truthful bidding, the Clock Stage ends when demand equals supply at a price of £100 per block, and since the BAFO Stage does not take place, Bidder C pays £700 for his 7 unpaired lots. However, Bidder C is better off ending the Clock Stage with excess supply when the price per block is less than £100, and then paying £400 for 7 unpaired lots in the BAFO Stage.

Thus truthful bidding can only be an equilibrium in these examples if: (i) the BAFO Stage always occurs; and (ii) BAFO Stage prices are not linked to the prices achieved in the Clock Stage.

Concluding comment on the BAFO Stage

What our examples indicate is that the BAFO Stage prices paid by individual bidders are not independent of their own bids - made in either the Clock or BAFO

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the last round of the Clock Stage, when there is no excess demand? Or is it the highest revenue obtained by taking into account all of the bids made in the Clock Stage (as suggested in Ausubel, Cramton and Milgrom, 2005)? In addition, what revenue should be attributed to blocks that remain unsold at the end of Clock Stage when there is excess supply (e.g zero, or the reserve prices)?

[18] For simplicity, we are ignoring the effect that reserve prices for the excess supply may have on the outcome of the Clock Stage, i.e. we assume that reserve prices are equal to zero.

[19] The Annex provides a similar example with a small amount of incomplete information, and shows that inefficiency may result.

stages - so incentives for sincere bidding can be compromised. The auction rules neither fully specify how final auction prices will be determined from the bids, nor necessarily prevent bidders from competing against their own prior bids. An incompletely specified pricing rule makes it difficult for bidders to predict the likely consequences of their bids, and hence to predict the bidding strategies of other bidders. The auction is then more likely to achieve an inefficient allocation by creating uncertainty and distorting bidding incentives.

This may argue in favour of using a pay-as-bid package auction (also called a 'menu auction') in the BAFO Stage, which although not efficient in general, at least determines auction prices unambiguously, and is well-understood by bidders.[20] There seems to be no *a priori* reason to assume that a pay-as-bid auction will produce less efficient outcomes in this context than the current second-price rule.[21] Another alternative would be to implement the type of the 'proxy' auction proposed in Ausubel, Cramton and Milgrom (2005). This seems likely to reduce some of the difficulties (e.g. it will determine final auction prices), if not eliminate them entirely.[22]

3.3 The Assignment Stage

The Assignment Stage of the auction seems to be poorly designed to achieve its stated objective of efficiently assigning particular lots to those who value them most. A first-price, sealed-bid auction is unlikely to achieve such an outcome. In addition, the effects of any uncertainty over final allocations created by the Assignment Stage will be felt in the auction's first two stages, since bidders will be uncertain about how they should value any particular unassigned lot. This will likely lead to some bidders bidding less aggressively than they otherwise would have in the earlier stages, further reducing the potential efficiency of the auction.[23]

Another issue is that even if the winning bidders' demands for frequency assignments are perfectly consistent, the proposed sealed-bid auction may induce bidders to effectively 'bid against themselves'. When no two bidders are interested in the same specific frequencies, they may still end up paying for assignments for which there was never any competition.

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[20] Two-stage 'Anglo-Dutch' auctions, with an initial clock stage, are currently being used to allocate virtual power plant capacity in Denmark for differing contract durations. See http://www.elsamvpp.com/multimedia/VirtualPowerPlant_Auctions1.pdf.

[21] Milgrom (2006a) compares pay-as-bid package auctions and proxy auctions. See also Brunner, Goeree, Holt, and Ledyard (2006) who have experimented with a two-stage Anglo-Dutch combinatorial auction, consisting of a multi-round clock stage followed by a single round of sealed bids, although their concern lay primarily in preventing collusive outcomes.

[22] See also Milgrom (2006b).

[23] For example, some types of bidder may care a great deal about the particular frequencies they will be assigned, while other types of bidder are relatively indifferent to this issue. The latter will then have an advantage in the earlier stages of the auction, since they will be more confident about bidding up to their relatively certain valuations, while the former type of bidder may decide to bid more cautiously in response to the uncertainty created by the Assignment Stage auction.

Moreover, for technological reasons, bidders may have different valuations for specific frequencies depending on the identities of the bidders to which adjacent frequencies are awarded, because of the "*significant potential for interference between neighbouring users*" (A8.29). This is especially likely to be the case if different bidders demand the same frequencies for different uses. In this context, to be able to bid efficiently, bidders may need to know the identities of the bidders to which all frequencies are assigned.

For these reasons, a more transparent process in the assignment stage would appear to be preferable. An obvious solution would be to hold a simultaneous, multi-round, ascending auction (SMRA) that reveals the identities of the highest bidders at the end of each round. Such an auction would provide more relevant information to bidders, and so potentially increase the efficiency of the spectrum assignment. And if bidders expect to be able to obtain an efficient assignment in the Assignment Stage, they are also more likely to bid efficiently in the earlier stages of the auction. For example, bidders who care a great deal about the particular frequencies they will be assigned will know that they will be able to obtain their preferred frequencies in the assignment stage, so long as they are willing to pay enough. With a single-round, sealed-bid auction on the other hand, this is far from guaranteed.

Another possibility would be to drop the Assignment Stage altogether by holding a multi-round clock auction with 38 clock prices in the first stage. Although this may sound like a large number, Ofcom's current proposals for the 1452-1492 MHz band already envisage a similar auction with 17 clock prices in each round, and the restrictions of contiguity, and on the number of paired lots available in the 2.6 GHz band, may reduce the complexity of a 38 clock auction somewhat.[24]

4. Conclusion

Ofcom's proposed auction for the 2.6 GHz and the 2010 MHz bands is a modified version of a 'clock-proxy' auction (as described in Ausubel, Cramton and Milgrom, 2005), with the 'proxy' stage replaced by a single-round, second-price package auction. To our knowledge, clock-proxy auctions have yet to be used in practice, and have not been subject to comprehensive laboratory testing. Other multi-round combinatorial auctions have at least been tested in laboratory settings (e.g. the FCC's simultaneous, multi-round auction with package bidding, and the Anglo-Dutch combinatorial auction),[25] and two-stage Anglo-Dutch auctions, with an initial clock phase followed by a single-round pay-as-bid auction, are currently being used to sell virtual power plant capacity in Denmark. Ofcom's consultation

[24] A potential drawback of this suggestion is that it may not be desirable to provide detailed information on 'neighbouring bidders' in such an auction, due to worries about bidder coordination or collusion, possibly reducing its efficiency. (However, Ofcom does not appear to be unduly concerned about this; see A8.16). In Ofcom's proposed Assignment Stage, worries about collusion no longer apply since an allocation has already been determined, so providing such detailed information raises no such problems.

[25] See Goeree, Holt and Ledyard (2006) and Brunner, Goeree, Holt, and Ledyard (2006) respectively.

document makes little or no reference to alternative auction designs, however, while aspects of its own auction proposals would appear to be open to question.

Given the size and importance of the auction for the 2.6 GHz and the 2010 MHz bands, we would suggest that a number of alternative auction designs be given further consideration, and that they be subject to laboratory testing. The types of auction mentioned immediately above could all be usefully considered. In addition, various versions of Ofcom's proposed auction design should be tested to identify potential flaws in advance of implementation. Current theory alone is not sufficient to determine the merits and demerits of the detailed auction rules in complex combinatorial settings.[26]

Some of the issues which should be addressed are:

1. The activity rules in the Clock Stage might be relaxed to allow bidders to switch demand between lot types subject to an appropriate eligibility rule, and possibly even to increase demand for one type of lot more than they reduce their demand for a different type of lot as the auction proceeds.
2. The BAFO Stage should be held irrespective of whether or not there is excess supply at the end of Clock Stage, or dropped altogether.
3. The rules on BAFO Stage bids should probably be modified so that:
    i. bidders make Capped BAFO bids only after the Clock Stage price discovery is complete; and
    ii. Capped BAFO bids are only restricted to be consistent with bids made during the Clock Stage, i.e. by the 'revealed preferences' of the bidders as determined from their Clock Stage bids.
4. The BAFO Stage 'second-price' rule does not fully specify how auction prices are determined from bids, and it does not eliminate incentives for insincere bidding and manipulation. Further consideration should be given to replacing it with a single-round, pay-as-bid auction, or a 'proxy' auction of the type proposed in the recent literature.
5. The rules which link the Clock Stage to the BAFO Stage also appear to give rise to incentives for price manipulation, and potentially conflict with the dual objectives of price discovery (in the Clock Stage) and efficiency (in the BAFO Stage).
6. The Assignment Stage single-round, pay-as-bid auction is likely to lead to inefficiency, and should either be replaced by an SMRA in which much more information is revealed to bidders, or dropped altogether in favour of a 38 clock auction in the first stage.

[26] Given that Ofcom is proposing to use a very similar auction design for the 1452-1492 spectrum band, the argument for laboratory testing is even stronger.

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Annex: Example of bidding with incomplete information

Ofcom's rule in A8.170 which links Clock Stage outcomes to prices in the BAFO Stage can create incentives for strategic bidding which lead to inefficiency when there is incomplete information. To see this, assume there are 7 blocks on sale and three bidders, and that Bidder A can be of two different types, A1 or A2. Suppose the bidder's preferences are given by:

1. Bidder A of Type A1 wants 2 unpaired lots for a maximum price of £200;
2. Bidder A of Type A2 wants 2 unpaired lots for a maximum price of £400;
3. Bidder B wants 1 paired lot for a maximum price of £200;
4. Bidder C wants 6 unpaired lots for a maximum price of £606.

Bidder C is unsure which type of Bidder A he is facing, and accords a 50% probability to each type.

Truthful bidding with no linkage

Assuming that each bidder bids truthfully, if Bidder A is of type A1, Bidder A and Bidder B drop out of the Clock Stage when the price per block reaches £100, at which point the Clock Stage ends with Bidder C bidding for 6 blocks, and the auction proceeds to the BAFO Stage. In the absence of the rule in A8.170, the BAFO Stage then awards 6 unpaired lots to Bidder C for a price of £400, the sum of the bids made by Bidder A1 and Bidder B.

If Bidder A is of type A2, the Clock Stage ends when Bidder C drops out at price £101 per block. The BAFO Stage then awards 6 unpaired lots to Bidder C for a price of £600, the sum of the bids made by Bidder A2 and Bidder B.

Note that without the price linkage between the Clock Stage and the BAFO Stage, it is always optimal for Bidder C to bid for 6 unpaired lots in the Clock Stage until his package bid is worth at least £601. If he dropped out any earlier, the activity rule would prevent him from bidding high enough to always win the auction in the BAFO Stage. Given this, Bidders A and B may as well bid truthfully also, so truthful bidding is an equilibrium.

Bidding with a price linkage

With truthful bidding, the 'outcome' of the Clock Stage when Bidder A is type A1 implies a minimum revenue of £600. Therefore, under Ofcom's linkage rule (A8.170), Bidder C would pay £600 for 6 unpaired lots in either of the above cases (i.e. regardless of Bidder A's type). Given this, assuming truthful bidding by Bidders A and B, Bidder C is better off not bidding truthfully and ending the Clock Stage earlier, when the price per block reaches £67, and submitting a "Capped BAFO bid" of £401 for 6 unpaired lots.[28] This allows Bidder C to win only if Bidder

[28] If Bidder C drops out of the Clock Stage at this price, the activity rule allows him to submit a "Capped BAFO bid" in the BAFO Stage of at most £402. Moreover, the 'outcome' of the Clock Stage implies a revenue of at most £396.

A is of type A1, but yields an expected profit of (£606-£401)½ = £102.5, which is higher than the profit Bidder C obtains from always winning the auction. Therefore, truthful bidding is not an equilibrium, and inefficiency results 50% of the time.

This example suggests that the linkage between Clock Stage and BAFO Stage prices makes it costly for bidders to reveal their preferences in the Clock Stage, since in the BAFO Stage they can find themselves competing against their own bids made in the Clock Stage. Activity rules may be sufficient to provide incentives for preference revelation in the Clock Stage while avoiding this problem.